(1)

$f(x) = ax^3 + bx^2 + cx$

$y = 10x - 5 \rightarrow x = 1 \quad y = 10 - 5 = 5 \quad A(1, 5)$

$f(1) = \boxed{a + b + c = 5}$

$f'(1) = 10 \rightarrow \left. 3ax^2 + 2bx + c \right|_{x=1} = 10$

$\boxed{3a + 2b + c = 10}$

$\int_0^1 f(x) = 7/4 \Rightarrow \left. \frac{ax^4}{4} + \frac{bx^3}{3} + \frac{cx^2}{2} \right|_0^1 = 7/4$

$\boxed{\frac{1}{4}a + \frac{1}{3}b + \frac{1}{2}c = \frac{7}{4}} \qquad \frac{3a + 4b + 6c}{12} = \frac{21}{12}$

$$\begin{cases} a + b + c = 5 & \rightarrow a = 5 - b - c \\ 3a + 2b + c = 10 & \rightarrow 3(5 - b - c) + 2b + c = 10 \\ 3a + 4b + 6c = 21 & \rightarrow 3(5 - b - c) + 4b + 6c = 21 \end{cases}$$

$15 - 3b - 3c + 2b + c = 10 \rightarrow -b - 2c = -5$

$\boxed{b + 2c = 5} \quad (1)$

$15 - 3b - 3c + 4b + 6c = 21 \rightarrow \boxed{b + 3c = 6} \quad (2)$

$(2) - (1) \rightarrow 0 + c = 1 \rightarrow c = 1$

$b = 5 - 2c \rightarrow b = 5 - 2 = 3 \quad \boxed{b = 3}$

$a = -b - c + 5 \rightarrow a = -3 - 1 + 5 \rightarrow \boxed{a = 1}$

$\boxed{f(x) = x^3 + 3x^2 + 1}$

2. $\int \ln^2 x \, dx \qquad t = \ln x \rightarrow x = e^t$

$dx = e^t dt$

$\int t^2 e^t dt$ integro per parti:

fattore differenziale: e^t

$= t^2 e^t - \int 2te^t dt = t^2e^t - \left[2te^t - \int 2e^t dt\right] =$

$= t^2e^t - 2te^t + 2e^t \cancel{+c} = e^t(t^2 - 2t + 2)$

$= x(\ln^2 x - 2\ln x + 2) + C$

---

3. $\int \frac{-2x^3-2}{x-2} dx$ Effettuo la divisione fra polinomi

$$
\begin{array}{l|l}
-2x^3 \qquad -2 & x-2 \\
\cline{2-2}
-2x^3 + 4x^2 & -2x^2-4x-8 \\
\cline{1-1}
= \quad -4x^2 - 2 & \\
\quad -4x^2 + 8x & \\
\cline{1-1}
\quad -8x - 2 & \\
\quad -8x + 16 & \\
\cline{1-1}
\quad -18 &
\end{array}
$$

Quindi

$-2x^3 - 2 = (-2x^2-4x-8)(x-2) - 18$

Verifica: $-2x^3 \underline{+4x^2} \underline{-4x^2} + 8x - 8x + 16 - 18 =$

$= -2x^3 - 2 \qquad$ c.v.d.

Quindi $\int \frac{-x^3-2}{x-2} dx = \int \frac{(-2x^2-4x-8)(x-2)-18}{x-2} dx =$

$\int \left[-2x^2-4x-8 - \frac{18}{x-2}\right] dx = -\frac{2x^3}{3} - \frac{4x^2}{2} - 8x - 18\ln|x-2| + c$

$F(x) = -\frac{2}{3}x^3 - 2x^2 - 8x - 18\ln|x-2|$

$$\int_{-\frac{\pi}{2}}^{\pi/2} \operatorname{sen}^2 x\, dx = \int_{-\pi/2}^{\pi/2} \operatorname{sen} x \cdot \operatorname{sen} x \cdot dx$$

Calcolo la primitiva integrando per parti:

$$\int \operatorname{sen} x \cdot \operatorname{sen} x\, dx = -\cos x \cdot \operatorname{sen} x - \int -\cos x \cdot \cos x\, dx \Rightarrow$$

$$= -\cos x \operatorname{sen} x + \int \cos^2 x\, dx \Rightarrow$$

$$= -\cos x \operatorname{sen} x + \int (1 - \operatorname{sen}^2) dx \Rightarrow$$

$$= -\cos x \operatorname{sen} x + \int 1 \cdot dx - \int \operatorname{sen}^2 dx \Rightarrow$$

$$\Rightarrow \int \operatorname{sen}^2 x\, dx + \int \operatorname{sen}^2 dx = -\cos x \operatorname{sen} x + x$$

$$\Rightarrow 2 \int \operatorname{sen}^2 dx = -\cos x \operatorname{sen} x + x$$

PRIMITIVA:

$$\Rightarrow \int \operatorname{sen}^2 dx = -\frac{\cos x \operatorname{sen} x}{2} + \frac{x}{2} = F(x)$$

ORA CALCOLO L'INTEGRALE DEFINITO $F\left(\frac{\pi}{2}\right) - F\left(-\frac{\pi}{2}\right)$

$$\Rightarrow \int_{-\pi/2}^{\pi/2} \operatorname{sen}^2 x\, dx = \frac{-\overset{=0}{\cos\frac{\pi}{2}} \operatorname{sen}\frac{\pi}{2} + \frac{\pi}{2}}{2} - \left[\frac{-\overset{=0}{\cos\left(-\frac{\pi}{2}\right)} \operatorname{sen}\left(-\frac{\pi}{2}\right) - \frac{\pi}{2}}{2}\right]$$

$$= \frac{\frac{\pi}{2}}{2} + \frac{\frac{\pi}{2}}{2} = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$$